arXiv:0704.3798v1 [q-fin.ST] 28 Apr 2007

Modeling the Epps effect of cross correlations in asset prices

Bence Tóth[a,b], Bálint Tóth[c], and János Kertész[b,d]

[a]ISI Foundation - Viale S. Severo, 65 - I-10133 Torino, Italy
[b]Institute of Physics, Budapest University of Technology and Economics - Budafoki út. 8. H-1111 Budapest, Hungary
[c]Institute of Mathematics, Budapest University of Technology and Economics - Egry József u. 1. H-1111 Budapest, Hungary
[d]Laboratory of Computational Engineering, Helsinki University of Technology - P.O.Box 9203, FI-02015, Finland

ABSTRACT

We review the decomposition method of stock return cross-correlations, presented previously[1] for studying the dependence of the correlation coefficient on the resolution of data (Epps effect). Through a toy model of random walk/Brownian motion and memoryless renewal process (i.e. Poisson point process) of observation times we show that in case of analytical treatability, by decomposing the correlations we get the exact result for the frequency dependence. We also demonstrate that our approach produces reasonable fitting of the dependence of correlations on the data resolution in case of empirical data. Our results indicate that the Epps phenomenon is a product of the finite time decay of lagged correlations of high resolution data, which does not scale with activity. The characteristic time is due to a human time scale, the time needed to react to news.

Keywords: Financial correlations, Epps effect, High frequency data, Market microstructure, Renewal process

1. INTRODUCTION

Stock return correlations decrease as the sampling frequency of data increases, as reported for the first time by Epps in 1979.[2] Since his discovery the phenomenon has been detected in several studies of different stock markets[3–5] and foreign exchange markets.[6,7]

The estimation of the asymptotic cross correlations between the individual assets is of major importance since these are the main factors in classical portfolio management. This is, however, hampered by the limited number of data. As high resulotion data are available in abundance, it is important to understand and give an accurate description of correlations for different sampling frequencies. This is especially so, as today the time scale in adjusting portfolios to relevant news may be in the order of minutes. Since its discovery, considerable effort has been devoted to uncover the phenomenon found by Epps.[8–13] Up to now two main factors causing the effect have been revealed: The first one is a possible lead-lag effect between stock returns[14–16] which appears mainly between stocks of very different capitalisation and if there is some functional dependence between them. In this case the maximum of the time-dependent correlation function is at non zero time lag, resulting in increasing correlations as the sampling time scale gets into the same order of magnitude as the characteristic lag. This factor can be easily understood, morever, in a recent study[16] we showed that through the years this effect becomes less important as the characteristic time lag shrinks, signalising an increasing efficiency of stock markets. It has to be emphasized that the Epps effect can also be found in the absence of the lead-lag effect, thus in the following we will focus only on other possible factors.

The second, more important factor is the asynchronicity of ticks in case of different stocks.[8,9,14,17] Empirical results[8] showed that taking into account only the synchronous ticks reduces to a great degree the Epps effect, i.e. measured correlations on short sampling time scale increase. Naturally one would expect that for a given sampling frequency growing activity decreases the asynchronicity, leading to a weaker Epps effect. Indeed Monte Carlo experiments showed an inverse relation between trading activity and the correlation drop.[8]

Bence Tóth: E-mail: bence@maxwell.phy.bme.hu

In our previous papers[1,18] we introduced a framework for describing the correlations on different time scales. We discussed the deficiencies of existing descriptions of the phenomenon, especially the fact that the characteristic time of the Epps effect does not scale with activity, thus can not be solely caused by the asynchronicity of ticks, and presented a decomposition process of the equal-time correlations on all time scales by writing them as functions of time dependent correlations on shorter time scales. We demonstrated the decomposition on a model case and showed fits for the Epps curves in case of real data, getting a good agreement with the measured correlations. In this paper we elaborate on the toy model[1] showing that the result through decomposing the correlations leads us to the exact solution.

In the following, first we summarize the decompostion of correlations written in details in our previous paper (Section 2). In Section 3 we show that the decomposition process leads to the exact analytic solution in a treatable model case. At the end of the paper (Section 4) we show an example of fitting the Epps curve for real stock data and review the process we believe to lie under the phenomenon.

2. DECOMPOSITION OF CORRELATIONS

We are interested in correlations between the logarithmic returns of stock prices as a function of the sampling time scale of data. The log-returns are defined by:

$$r^A_{\Delta t}(t) = \ln \frac{p^A(t)}{p^A(t-\Delta t)}, \tag{1}$$

where $p^A(t)$ stands for the price of stock A at time t. Throughout the paper we will assume that the return distributions are stationary both empirically and in the model. The time dependent correlation function $C^{A/B}_{\Delta t}(\tau)$ of stocks A and B is defined by

$$C^{A/B}_{\Delta t}(\tau) = \frac{\left\langle r^A_{\Delta t}(t) r^B_{\Delta t}(t+\tau)\right\rangle - \left\langle r^A_{\Delta t}(t)\right\rangle \left\langle r^B_{\Delta t}(t+\tau)\right\rangle}{\sigma^A \sigma^B}. \tag{2}$$

The notation $\langle \cdots \rangle$ stands for the moving time average over the considered period:

$$\langle r_{\Delta t}(t)\rangle = \frac{1}{T-\Delta t} \sum_{i=\Delta t}^{T} r_{\Delta t}(i), \tag{3}$$

where time is measured in seconds and T is the time span of the data. The standard deviation σ of the returns is:

$$\sigma = \sqrt{\langle r_{\Delta t}(t)^2\rangle - \langle r_{\Delta t}(t)\rangle^2}, \tag{4}$$

both for A and B in Equation 2. The equal-time correlation coefficient is naturally: $\rho^{A/B}_{\Delta t} \equiv C^{A/B}_{\Delta t}(\tau = 0)$.

Using the property that returns in a certain time window Δt are mere sums of returns in smaller, non-overlapping windows Δt_0, where Δt is a multiple of Δt_0 and assuming the time average of stock returns to be zero, we are able to deduce the following relationship between correlations on different time scales (for details see Ref. 1):

$$\begin{aligned}
\rho^{A/B}_{\Delta t} = &\left(\sum_{x=-\frac{\Delta t}{\Delta t_0}+1}^{\frac{\Delta t}{\Delta t_0}-1} \left(\frac{\Delta t}{\Delta t_0} - |x|\right) f^{A/B}_{\Delta t_0}(x\Delta t_0)\right) \times \\
&\left(\sum_{x=-\frac{\Delta t}{\Delta t_0}+1}^{\frac{\Delta t}{\Delta t_0}-1} \left(\frac{\Delta t}{\Delta t_0} - |x|\right) f^{A/A}_{\Delta t_0}(x\Delta t_0)\right)^{-1/2} \times \\
&\left(\sum_{x=-\frac{\Delta t}{\Delta t_0}+1}^{\frac{\Delta t}{\Delta t_0}-1} \left(\frac{\Delta t}{\Delta t_0} - |x|\right) f^{B/B}_{\Delta t_0}(x\Delta t_0)\right)^{-1/2} \rho^{A/B}_{\Delta t_0}.
\end{aligned} \tag{5}$$

.

In Equation 5 $f^{A/B}_{\Delta t_0}(x\Delta t_0)$, $f^{A/A}_{\Delta t_0}(x\Delta t_0)$ and $f^{B/B}_{\Delta t_0}(x\Delta t_0)$ are the decay functions of lagged correlations on the short time scale (Δt_0) given by the expression

$$f^{A/B}_{\Delta t_0}(x\Delta t_0) = \frac{\left\langle r^A_{\Delta t_0}(t) r^B_{\Delta t_0}(t + x\Delta t_0)\right\rangle}{\left\langle r^A_{\Delta t_0}(t) r^B_{\Delta t_0}(t)\right\rangle}, \tag{6}$$

(and similarly for $f^{A/A}_{\Delta t_0}(x\Delta t_0)$ and $f^{B/B}_{\Delta t_0}(x\Delta t_0)$), defined for both positive and negative x values.

This way we obtained an expression of the correlation coefficient for any sampling time scale, Δt, by knowing the coefficient on a shorter sampling time scale, Δt_0, and the decay of lagged correlations on the same shorter sampling time scale (given that Δt is multiple of Δt_0). Our method is to measure the correlations and fit their decay functions on a certain short time scale and compute the Epps curve using the above formula.

3. ANALYTICALLY TREATABLE CASE

In this section we demonstrate that the solution through the decomposition of the correlations leads to the exact solution in case of analytical tretability of the decay functions. First we discuss a toy model describing two correlated but asynchronous time series, then we show that the two ways of deducing expressions for the relation of the correlations on different time scales lead to the same result.

3.1. The model

We would like to study generated time series which have similar properties as real world price time series. To do this, we simulate two correlated but asynchronous logarithmic price time series. As a first step we generate a core random walk with unit steps up or down in each second with equal possibility ($W(t)$). Second we sample the random walk, $W(t)$, twice independently with waiting times drawn from an exponential distribution. This way we obtain two time series ($\log p^A(t)$ and $\log p^B(t)$), which are correlated since they are sampled from the same core random walk, but the steps in the two walks are asynchronous. The core random walk is:

$$W(t) = W(t-1) + \varepsilon(t), \tag{7}$$

where $\varepsilon(t)$ is ± 1 with equal probability (and $W(0)$ is set high in order to avoid negative values). We define the steps occuring in the two asynchronous random walks respectively as $\underline{\omega}^A = \{\omega^A_i\}$ and $\underline{\omega}^B = \{\omega^B_i\}$ being two Poisson point processes on $\mathbb{R}^+$ with density λ, thus the time increments are drawn from the exponential distribution:

$$P(y) = \begin{cases} \lambda e^{-\lambda y} & \text{if } y \geq 0 \\ 0 & y < 0 \end{cases} \tag{8}$$

with parameter $\lambda = 1/60$. Between two consecutive steps the sampling walkers do not move, thus:

$$\begin{aligned} \gamma^A(t) &:= max\{\omega^A_i : \omega^A_i < t\} \\ \gamma^B(t) &:= max\{\omega^B_i : \omega^B_i < t\} \end{aligned} \tag{9}$$

and the two walks become:

$$
\begin{aligned}
\log p^A(t) &:= W\big(\gamma^A(t)\big) \\
\log p^B(t) &:= W\big(\gamma^B(t)\big)
\end{aligned}
\tag{10}
$$

A snapshot as an example of the generated time series with exponentially distributed waiting times can be seen on Figure 1.

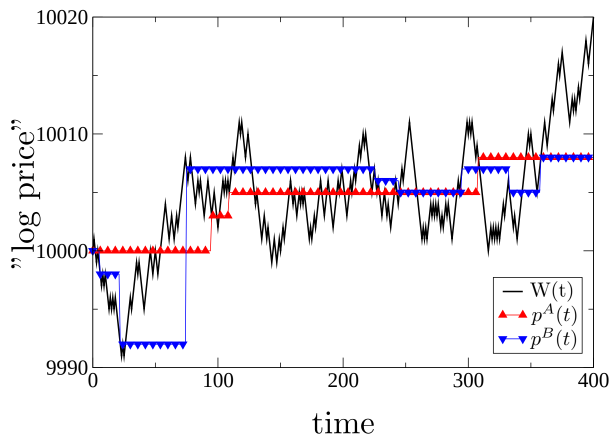

Figure 1. A snapshot of the model with exponentially distributed waiting times. The original random walk is shown with lines (black), the two sampled series (the log prices) with dots and lines (red) and triangles and lines (blue).

As a next step we create the return time series ($r^A_{\Delta t}(t)$ and $r^B_{\Delta t}(t)$) of $\log p^A(t)$ and $\log p^B(t)$, and study their cross-correlation as a function of sampling time scale. In the model case we set the smallest time scale $\Delta t_0 = 1$ time step.

3.2. Decomposing the correlations in the model

Having a random walk model, the autocorrelation function of the steps is zero for all non-zero time lags:

$$
f^{A/A}_{\Delta t_0}(x\Delta t_0) = f^{B/B}_{\Delta t_0}(x\Delta t_0) = \delta_{x,0}. \tag{11}
$$

For the case when steps in the random walks are sparse in time, thus when $\lambda\Delta t_0 \ll 1$, the decay function is an exponential decay (see Figure 2):

$$
f^{A/B}_{\Delta t_0}(x\Delta t_0) = e^{-\lambda\Delta t_0|x|}, \tag{12}
$$

with the same parameter as the original Poisson process in Equation 8.

Thus the ratio of the correlations can be written in the following way:

$$
\frac{\rho^{A/B}_{\Delta t}}{\rho^{A/B}_{\Delta t_0}} = \frac{\Delta t_0}{\Delta t} \sum_{x=-\frac{\Delta t}{\Delta t_0}+1}^{\frac{\Delta t}{\Delta t_0}-1} \left[\left(\frac{\Delta t}{\Delta t_0} - |x| \right) e^{-\lambda\Delta t_0|x|} \right] =
$$

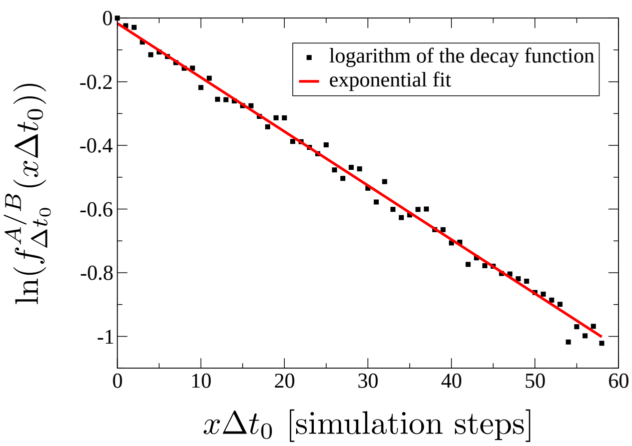

Figure 2. The logarithm of the decay function and its exponential decay fit on a log-lin scale. The parameter of the exponential decay is 59.1, very near to the parameter of the original exponential distribution of the waiting times.

$$
\begin{aligned}
&= \frac{\Delta t_0}{\Delta t}\left[\frac{\Delta t}{\Delta t_0} + 2\sum_{x=1}^{\frac{\Delta t}{\Delta t_0}-1}\left(\frac{\Delta t}{\Delta t_0} - x\right)e^{-\lambda \Delta t_0 x}\right] \\
&= 1 + 2\sum_{x=1}^{\frac{\Delta t}{\Delta t_0}-1} e^{-\lambda\Delta t_0 x} - 2\frac{\Delta t_0}{\Delta t}\sum_{x=1}^{\frac{\Delta t}{\Delta t_0}-1} x e^{-\lambda \Delta t_0 x}.
\end{aligned} \tag{13}
$$

The first sum on the right side of Equation 13 is the sum of a geometric series and can be written in a closed form in the following way:

$$\sum_{x=1}^{\frac{\Delta t}{\Delta t_0}-1} e^{-\lambda \Delta t_0 x} = \frac{e^{-\lambda\Delta t_0} - e^{-\lambda \Delta t}}{1 - e^{-\lambda \Delta t_0}}. \tag{14}$$

Using the Taylor expansion of the exponential function:

$$e^y = \sum_{n=0}^{\infty} \frac{y^n}{n!}, \tag{15}$$

and applying that $\lambda\Delta t_0 \ll 1$, we can neglect the high order terms in the sum in Equation 15 and take into account only the terms up to linear order in $\lambda\Delta t_0$. Hence

$$\sum_{x=1}^{\frac{\Delta t}{\Delta t_0}-1} e^{-\lambda \Delta t_0 x} \approx \frac{1 - \lambda\Delta t_0 - e^{-\lambda\Delta t}}{\lambda \Delta t_0}. \tag{16}$$

The second sum on the right side of Equation 13 can be obtained by differentiating 14 and taking the small $\lambda\Delta t_0$ limit:

$$\sum_{x=1}^{\frac{\Delta t}{\Delta t_0}-1} x e^{-\lambda \Delta t_0 x} \approx \frac{1 - \lambda\Delta t_0 + \left[-\lambda\Delta t - 1 + \lambda\Delta t_0\right] e^{-\lambda \Delta t}}{(\lambda\Delta t_0)^2}. \tag{17}$$

Inserting Equation 16 and quation 17 into Equation 13 we get:

$$\begin{aligned}\frac{\rho_{\Delta t}^{A/B}}{\rho_{\Delta t_0}^{A/B}} \approx 1 + \frac{2 - 2\lambda\Delta t_0 - 2e^{-\lambda\Delta t}}{\lambda\Delta t_0} - \\ -\frac{2\Delta t_0}{\Delta t}\frac{1 - \lambda\Delta t_0 + \left[-\lambda\Delta t - 1 + \lambda\Delta t_0\right] e^{-\lambda\Delta t}}{(\lambda\Delta t_0)^2} = \\ = \frac{1}{(\lambda\Delta t_0)^2}\left[-(\lambda\Delta t_0)^2 + 2\lambda\Delta t_0 - \frac{2\Delta t_0}{\Delta t} + \frac{2\lambda\Delta t_0^2}{\Delta t}\right] + \\ \frac{1}{(\lambda\Delta t_0)^2} e^{-\lambda\Delta t}\left(\frac{2\Delta t_0}{\Delta t} - \frac{2\lambda\Delta t_0^2}{\Delta t}\right).\end{aligned} \tag{18}$$

Since $(\lambda\Delta t_0)^2$ and $2\lambda\Delta t_0^2/\Delta t$ is much smaller than the other expressions appearing in the denominator of Equation 18, we can neglect them. Hence the final relation becomes

$$\frac{\rho_{\Delta t}^{A/B}}{\rho_{\Delta t_0}^{A/B}} \approx \frac{2}{\lambda\Delta t_0} + \frac{2}{\lambda^2\Delta t\Delta t_0}\left(e^{-\lambda\Delta t} - 1\right). \tag{19}$$

3.3. The exact analytical solution

For the case described above the correlation can be given in an exact analytical form using sepcial properties of the Poisson processes. We go to a conrinuous description and use a Brownian motion instead of a discrete random walk. We have:

$$\left\langle r_{\Delta t}^{A}(t)\right\rangle = \left\langle r_{\Delta t}^{B}(t)\right\rangle = 0 \tag{20}$$

and

$$\left\langle (r_{\Delta t}^{A}(t))^2\right\rangle = \left\langle (r_{\Delta t}^{B}(t))^2\right\rangle = \Delta t. \tag{21}$$

The interesting part of the correlation is the average of the cross-product of the two returns, which is the following:

$$\begin{aligned}\left\langle r_{\Delta t}^{A}(t) r_{\Delta t}^{B}(t))\right\rangle = \\ = \mathbb{E}\left(\mathbb{E}\left(\left(W(\gamma^A(t)) - W(\gamma^A(t-\Delta t))\right)\left(W(\gamma^B(t)) - W(\gamma^B(t-\Delta t))\right)\middle| \begin{array}{c}\underline{\omega}^A \\ \underline{\omega}^B\end{array}\right)\right),\end{aligned} \tag{22}$$

where the inner expectation averages with $\underline{\omega}^A$ and $\underline{\omega}^B$ being given, while the outer expectation averages over $\underline{\omega}^A$ and $\underline{\omega}^B$. Equation 22 can be rewritten as the expectation of the intersection of time intervals between the last step before time t and the last step before time $(t - \Delta t)$ for the two walks respectively:

$$\left\langle r_{\Delta t}^{A}(t) r_{\Delta t}^{B}(t))\right\rangle = \mathbb{E}\left(\left|\left[\gamma^A(t-\Delta t), \gamma^A(t)\right] \cap \left[\gamma^B(t-\Delta t), \gamma^B(t)\right]\right|\right). \tag{23}$$

To detemine the expression in Equation 23 we need to know the probability distribution of the minimum and the maximum of two independently and exponentially distributed variables. Let ξ and η be such. Then

$$
\mathbb{P}\big(min\{\xi,\eta\} \in (x, x+dx)\big) = 2\lambda e^{-2\lambda x} dx
$$
$$
\mathbb{P}\big(max\{\xi,\eta\} \in (x, x+dx)\big) = 2\lambda (e^{-\lambda x} - e^{-2\lambda x}) dx. \tag{24}
$$

Thus the correlation coefficient becomes:

$$
\rho^{A,B}_{\Delta t} = \frac{2}{\lambda \Delta t} \int_0^{\lambda \Delta t} \big(\lambda \Delta t - x + \frac{1}{2}\big)\big(e^{-x} - e^{-2x}\big) dx =
$$
$$
= \frac{1}{\lambda \Delta t}\big(e^{-\lambda \Delta t} - 1\big) + 1. \tag{25}
$$

The ratio between the correlation coefficient on the sampling scale Δt and sampling scale Δt_0 is

$$
\frac{\rho^{A,B}_{\Delta t}}{\rho^{A,B}_{\Delta t_0}} = \frac{\frac{1}{\lambda \Delta t}\big(e^{-\lambda \Delta t} - 1\big) + 1}{\frac{1}{\lambda \Delta t_0}\big(e^{-\lambda \Delta t_0} - 1\big) + 1}, \tag{26}
$$

which in the $\lambda \Delta t_0 \ll 1$ limit follows as

$$
\frac{\rho^{A,B}_{\Delta t}}{\rho^{A,B}_{\Delta t_0}} = \frac{2}{\lambda \Delta t_0} + \frac{2}{\lambda^2 \Delta t \Delta t_0}\big(e^{-\lambda \Delta t} - 1\big). \tag{27}
$$

Hence we end up with exactly the same expression as deduced through the decomposition process in Equation 19.

4. RESULTS FOR STOCK DATA

With the results derived in the last section we showed for a case when the correlation can be computed analytically that our approach reproduces the exact solution. After this we show an example of fitting the measured correlation on real world data with the method of decomposing the correlation coefficient. More examples and details can be found in Ref. 1.

In the analysis of real world data we used the Trade and Quote (TAQ) Database of the New York Stock Exchange (NYSE) for the period of 4.1.1993 to 31.12.2003, containing tick-by-tick data. To avoid problems occurring from splits in the prices of stocks, which cause large logarithmic return values in the time series, we applied a filtering procedure. In high-frequency data, we omitted returns larger than the 5% of the current price of the stock. This retains all logarithmic returns caused by simple changes in prices but excludes splits which are usually half or one third of the price. We computed correlations for each day separately and averaged over the set of days, this way avoiding large overnight returns and trades out of the market opening hours.

To avoid new parameters in the model we use the raw decay functions in Equation 5, without fitting them. Since it is an empirical approach to determine the decay functions for real data, we have to distinguish the signal from the noise in the decay functions. According to this we use the decay functions for correlations only for short time lags. For the decay of the cross-correlations we take into account the function only up to the time lag where the decaying signal reaches zero for the first time, for larger lags we assume it to be zero. For the decay of autocorrelations consider the functions only up to the time lag where after the negative overshoot at the beginning they reach to zero from below for the first time, for larger lags we again define them as zero. In case of all stock pairs studied we found the decay functions disappearing after 5–15 minutes. In the empirical decays measured, Δt_0 is set to 2 minutes. Figure 3 shows the measured and the analytically computed Epps curves for the stockpair Merck & Co., Inc. (MRK) / Johnson & Johnson (JNJ), giving good agreement between the measured and computed coefficients.

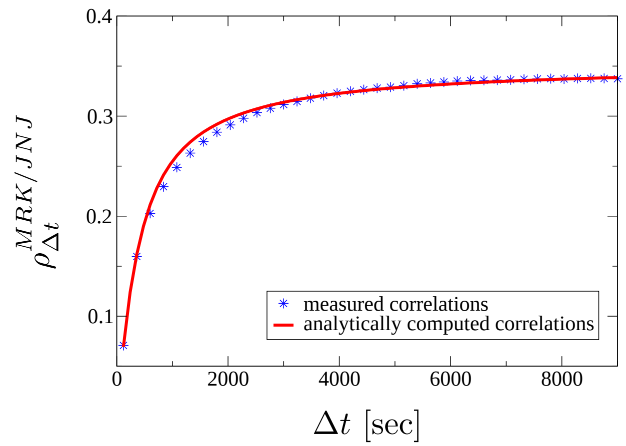

Figure 3. The measured and the analytically computed correlation coefficients as a function of sampling time scale for the pair: MRK/JNJ. Note that using only the correlations measured on the smallest time scale ($\Delta t_0 = 120$ seconds) we are able to give reasonable fits to the correlations on all time scales.

One can see, that the fits are able to describe the change of correlation with increasing sampling time scale. Through the decomposition process of the correlations in Equation 5 we can see that the important property that causes the Epps effect is the finite decay of correlations on the high resolution scale (Δt_0). If these decays were very prompt, the Epps phenomenon would disappear after a few seconds or minutes. This finite decay of the correlations on the short time scale (Δt_0) is a consequence of the market microstructure. Reaction to a certain piece of news is usually spread out on an interval of a few minutes for the stocks[19, 20] due to human trading nature, thus not scaling with activity, with ticks being distributed more or less randomly. This means that correlated returns are spread out for this interval (asynchronously), causing non zero lagged correlations on the short time scale and thus the Epps effect. This way, as stated by Ref. 8, the asynchronicity is indeed important in describing the Epps effect but only in promoting the lagged correlations. Even in case of completely synchronous, but randomly spread ticks we could have the finite decay of lagged correlations on short time scale, and hence the Epps effect.

ACKNOWLEDGMENTS

Support by OTKA T049238 and OTKA K60708 is acknowledged.

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